

## First-Order Circuits

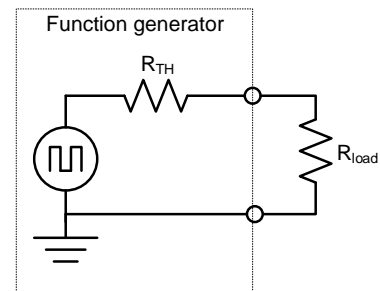
### Objectives

- 1) Gain an intuitive understanding of the concept of a time constant
- 2) Practice and learn new oscilloscope skills
- 3) Gain an intuitive understanding of the step response of first order (RC and RL) circuits
- 4) Determine the Thevenin resistance of the function generator
- 5) Learn how to create and analyze time-varying circuits in LTspice

### Prelaboratory

---

1. Consider the circuit of the figure to the right that shows a function generator model (inside the dotted box) connected to a load resistor. Students often wonder about the Thevenin resistor model  $R_{TH}$  inside the function generator model. What is it, and what is its purpose? You know intuitively that if that load resistor was replaced by a short, an infinite amount of current would not flow in the real-world; if it could, the function



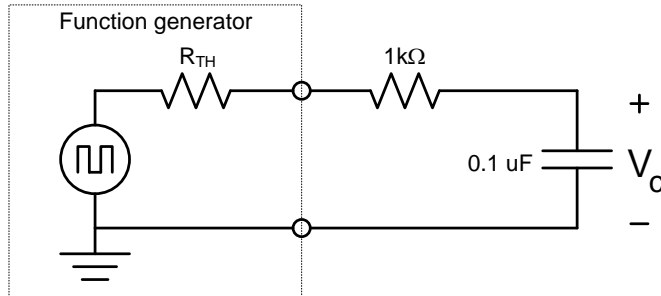
generator could run a city. Say you measure the actual amount of current that flows when the output is shorted and find it is 10mA. If the voltage source was 1V before the short was applied, then  $R_{TH}$  explains why an infinite current does not when it is shorted; it would be modeled as a  $100\Omega$  resistor so that 1V would produce 10mA when shorted. The  $R_{TH}$  is not an actual resistor inside the source; it simply models the unfortunate reality that a real-world source can't supply an infinite amount of current.

Assume the output resistance of the function generator ( $R_{TH}$  in the figure) is  $50\Omega$ . You could use an oscilloscope to set the function generator to create a  $4V_{pp}$  (Volts peak-to-peak, meaning  $\pm 2V$ , also called  $2V_{peak}$  or simply  $2V_p$ ) squarewave with no load resistor attached.

- a. **If you attach 1 k $\Omega$  load resistor across the function generator, what will the new voltage  $V_{pp}$  be across the load resistor? (It is just a voltage divider.)**
- b. **How about if you use a 10 $\Omega$  resistor instead?**

Moral of the story:

- a. As a general rule, as long as the load resistance is at least ten times the source resistance, you can think of the function generator as a perfect voltage source and ignore the source resistance.
- b. Don't ignore the Thevenin equivalent model of your test instruments...if your load resistance is less than 10 times the instrument's Thevenin equivalent, you need to know!



2. Now add a capacitor in series with the circuit as shown above. **Calculate the period of the squarewave necessary to be about 10 times longer than the RC time constant of the circuit** (remember to include the function generator's  $R_{TH}$  on the RC time constant). You can round the result; what is important is that each leg of the squarewave (positive and negative) will be about 5 times longer than the circuit's time constant, so the exponentially-decaying transients on each squarewave transition will have nearly completely decayed by the time the next squarewave transition occurs.

3. **Use the transient analysis on LTspice to determine the voltage  $v_c(t)$  over two periods of the squarewave generator.** Use a symmetric  $4V_{pp}$  ( $\pm 2V$  or  $2V_p$ ) square wave and print a graph of the voltage across the capacitor over two complete cycles of the squarewave generator. (You'll have to experiment a little with this; don't wait until the day of the lab!) Not sure how to make a squarewave? Look at the transient analysis section of my LTspice Walkthrough if you don't remember how to do this.

There are many types of analyses that may be applied to circuits. In EE122 you worked with non-switched DC sources; in LTspice the *DC Operating Point Analysis* type can find the steady-state voltages and currents everywhere in such a circuit. Now in EE223 you need to analyze circuits with sources that are not constant; this analysis type is called *Transient Analysis*. To do this you must know the end time of your simulation. In the prelab step 2 (above) you determined the frequency of the squarewave; instruct LTspice to analyze between  $t=0$  and whatever time is necessary to view two complete squarewave cycles.

In summary, for the prelab, read the entire prelab and lab, and answer the bolded questions in the prelab. As always, the prelab is to be done individually, not with your lab partner. The first sentence, read and understand the entire lab, is critical. It will take some time and energy to do, but if you do not you will be lost in the lab even if you answer the prelab questions.

## Laboratory

1. **Determine the Thevenin resistance of the function generator.** You could theoretically short the output and measure the current that flows (i.e. measure  $V_{oc}$  and  $I_{sc}$ , as you learned in EE122) but shorting the output of voltage sources can break them. Instead, the standard way of measuring the Thevenin resistance involves not shorting the output, but instead putting in different load resistors until the voltage across the load is dropped by half. Then, by the voltage divider equation, the load resistor must be equal to the Thevenin internal source resistance.

**Important: Do not use the Agilent oscilloscope's peak-to-peak measurement—it will fluctuate wildly due to its inability to trigger. Instead, press the “meas” button and add the “Amplitude” measurement using the “Type” and “Add Measurement” softkeys on the left.**

Use a *decade resistance box* (they are the small beige boxes with the thumbwheel counters in the lab drawer) as the variable resistor. First, set the function generator to deliver a 1kHz square wave at  $V_{pp}=4V$  with the variable resistor set to a much higher resistance than the function generator (say,  $1k\Omega$ ) as shown in Figure 3 below. Make sure you are using scope probes and make sure that the oscilloscope's probe tip is set to 10:1 and is on DC coupling (options displayed by pressing the “1” button over the yellow background, short for “Channel 1”). Place the probe's leads to measure the voltage across the decade resistance box.

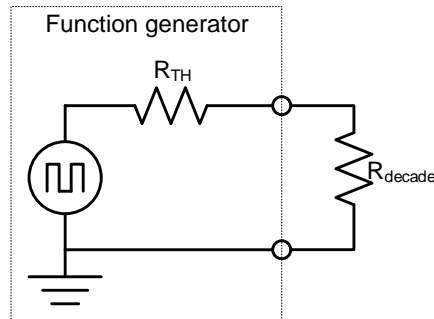


Figure 3: Circuit to determine the Thevenin equivalent resistance of the function generator. Note that the circuit is a *voltage divider*.

Next, lower the resistance using the thumbwheel on the box until the scope shows the square wave's amplitude has been cut in half, from  $\pm 2V$  to  $\pm 1V$ . This means half the voltage is being dropped by the internal resistance of the function generator, and the other half by the external variable resistor, so the two resistances must be equal. Now simply read the resistance dialed on the decade resistance box; that is the function generator's  $R_{TH}$ .

2. **Measure Vc for the first order circuit of Figure 4** (next page). To do this, the decade box and build the Figure 4 circuit using a normal resistor and capacitor on a protoboard. Use two scope probes:

- one scope probe is on channel 1, and measures the input between terminals A and C
- one scope probe is on channel 2 and measures the output between terminals B and C

If necessary, use AUTOSCALE to get an initial display. Set the function generator to output a squarewave of 1kHz, with a  $V_p = 2V$  ( $V_{pp}=4V$ ) with no voltage offset. Then record the actual waveform and component values in the table on the next page. Note: measure  $V_p$  and frequency using the scope; the markings on the function generator are only approximate.

Measurement	Nominal Value	Actual Value
$V_p$	2.00V	
f	1.00kHz	
R (resistance)	1k $\Omega$	
C (capacitance)	0.1 $\mu$ F	

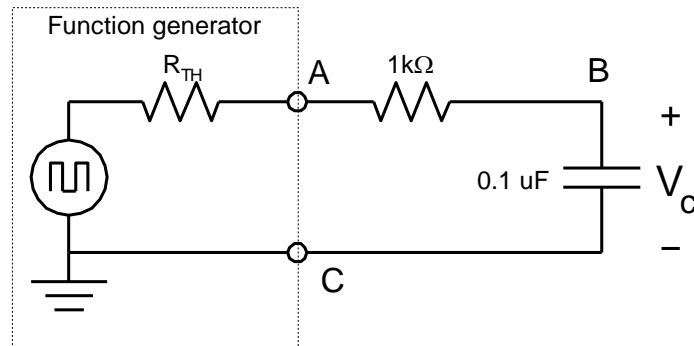


Figure 4: The first order circuit. The entire lab consists of finding the voltage across the resistor, the capacitor, and the current through the circuit. Limitations of the oscilloscope make this challenging.

In your lab notebook, record one complete cycle of  $V_c$ . It should look like an ocean wave:

To make this easier, change the scope horizontal control to view one cycle only, adjust the vertical and horizontal position controls so that the decaying exponential begins at the far left edge of the screen and adjust the vertical scale so the waveform fills the screen. These controls are highlighted below (note: the waveform in the image below is not from this lab!). **Draw  $v_c(t)$  in your lab notebook. Be quantitative;** label the horizontal and vertical axes, and any key information (e.g. max/min waveform height in Volts, time between peaks in milliseconds, how long it takes to decay, ...).

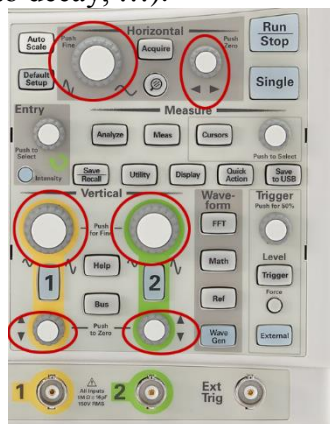


Figure 5: Oscilloscope controls highlighted that control the horizontal zoom (top left), horizontal offset (top right), vertical zoom (vertical middle), and vertical offset (bottom).

3. **Measure the current through the capacitor.** To measure the current through the capacitor you must get creative since the scope only measures voltage. To do this, note that the current through the capacitor is the same as the current through the resistor, and that the current through the resistor generates a proportional voltage across the resistor. So the voltage waveform across the resistor (across points AB) is the same as the current waveform through the capacitor, to within a scaling factor found by Ohm's Law. Determine that scaling factor. Then, draw the voltage waveform across the capacitor, and use your scaling factor to change the horizontal axis so it now represents the current.

Sounds easy, but you to measure the voltage across the resistor you CANNOT connect the positive probe tip to point A and the ground lead to point B in the diagram below, since inside the scope the ground lead is connected to point C and you will connect both sides of the capacitor to ground, shorting it out, and the circuit will no longer be the first order circuit you expect.

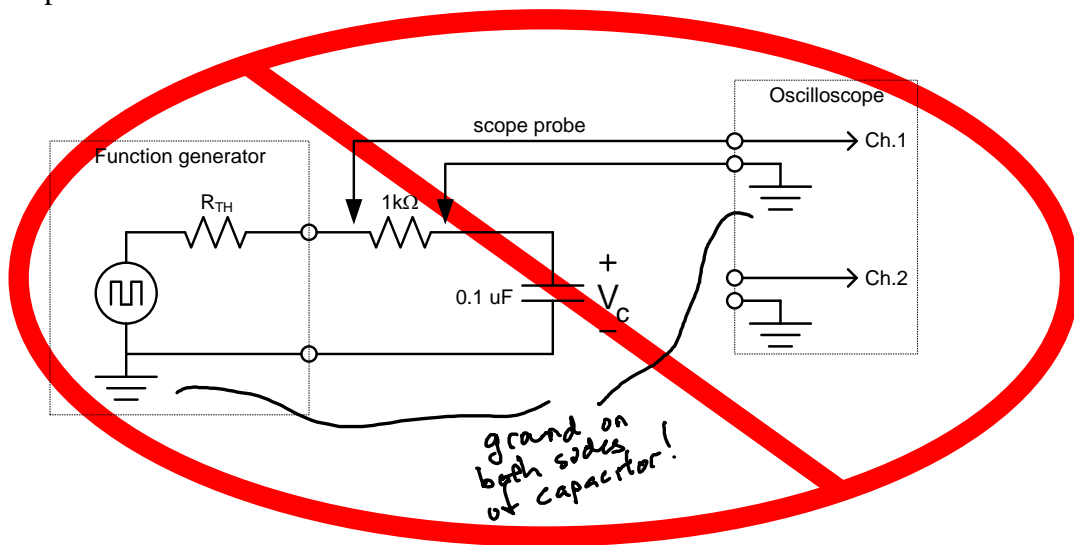


Figure 5- How **not** to connect the circuit to measure the voltage across the resistor. This violates the rule: "always connect the scope probe's ground lead to ground."

To fix this problem, you must use a new feature of the scope, the Math function, and use a second probe as explained on the following page.

The math function of the scope lets you subtract one channel from another. Since it uses two channels, it requires two scope probes, as shown below:

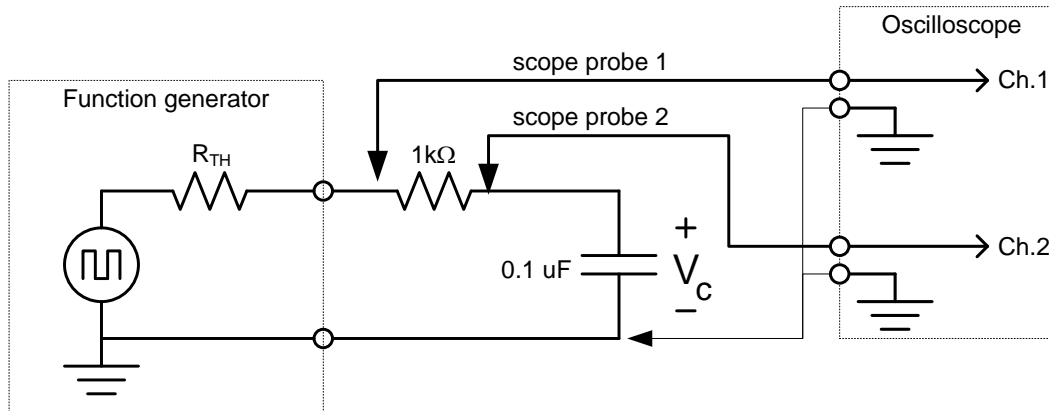
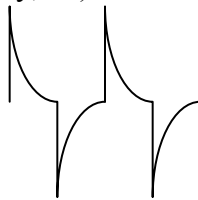


Figure 6: The correct way to use a scope to view the voltage waveform across the resistor. It requires two channels, but now the scope ground leads are both connected to ground. To view the voltage waveform, use the scope's Math function to subtract Channel 2 from Channel 1. **Make sure all grounds are connected.**

Connect Channel 1 to one side of the resistor, Channel 2 to measure the other side, and both channels' ground leads to ground (as they always must be). Use the "Math" function on the scope to find the difference of the signals (use the softkey on the left and change the "operator" to subtract). The result is the voltage difference across the resistor. In addition, you **must** have channel 1 and channel 2 have the same V per division in order to have an accurate reading using the math feature.

**Draw  $v_R(t)$  in your lab notebook. Be quantitative;** label the horizontal and vertical axes, and any key information (e.g. max/min waveform height in Volts, time between peaks in milliseconds, how long it takes to decay, ...). It should look roughly like this:

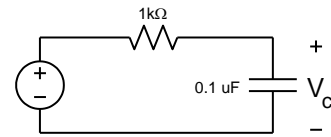


**Draw  $i_C(t)$  in your lab notebook.** Notice that this is the same shape as the graph you just drew for  $v_R(t)$  but with a different vertical axis scaling. Label it as above, but using height values of mA.

## Discussion Questions:

1. You can regard the function generator as a perfect voltage source if the load resistance is at least ten times higher than the source resistance. You wish to test your car's audio amplifier with a signal from the function generator. The input resistance of the audio amplifier is  $10\text{k}\Omega$ .
  - a) Will the audio amplifier load (i.e. change) the function generator's output when connected?
  - b) How about if you connect an  $8\Omega$  speaker to the function generator?
  - c) What could you put between the function generator and speaker to correct this?

2. In the prelab you analyzed the first-order RC circuit using LTspice. In the lab you built it and took experimental measures that should have agreed with your LTspice simulations. Now analyze it one more way: analytically, by deriving the  $v_c(t)$  equation using the plug and chug formula  $v_c(t) = V_\infty + (V_0 - V_\infty)e^{-t/\tau}$ . To do this, just analyze the circuit over one charging cycle rather than multiple periods. Specifically, using the schematic to the right, assume the capacitor is initially charged to  $-2\text{V}$  at  $t=0$  and at that time the function generator becomes  $+2\text{V}$ . Solve for  $v_c$  and graph this using Matlab, and attach a printout of the solution to your lab report. It should look like the solution you created in LTspice in your prelab. Hint: to plot  $v_c(t) = 3e^{-1000t}$  (not the right answer) over 1 millisecond you could enter



```
t=linspace(0,0.001, 500); % vector of 500 points b/n 0 and 0.001
v=3*exp(-1000*t); % creates the vector v
plot(t,v) % plots the result
```

Use xlabel, ylabel, and title to label your plot. If unsure about these functions, type “help plot” at the Matlab prompt.

3. If you wanted to make the system decay 10 times more slowly and could only change the resistor, what resistance would you choose?

In your report include the sketches you made from the oscilloscope ( $v_c$ ,  $v_r$ ,  $i_c$ ), the Matlab plot, and the LTspice results you obtained in the prelab. Examine possible reasons for disagreement between your experimental results, LTspice results, and Matlab plot. **Be quantitative!** For example, don't say “looks similar”, but instead “the rise time of the simulation is 4.5% greater than the actual rise time, which follows from the measured resistor tolerance...”